AP Calculus BC 2020-2021
Summer Assignments

Below is a list of problems that you need to be complete by the first day of school. I realize that you may need some help with these problems, and I am willing to answer your questions through email after you have put forth an effort to find the solution on your own. I will be checking my email periodically throughout the summer. I also would encourage you work these problems the last week before you return to school, not before then. You will need this information fresh in your minds when you return to school.

Chapter 0 Review Problems
Pages 55 – 57
Problem Numbers: 1, 3, 5-7, 8, 10, 12, 15-25 odd, 27-38, 39-44, 45, 47, 57-59, 62-65, and 68-70.

Chapter 1 Review Problems
Pages 101 – 103
Problem Numbers: 1-37 odd, 38, 39, 41, 51, 55-57.

***** Please neatly work the problems on separate sheets of paper**.***

Each problem should include all your work along with your solutions. When necessary, you need to write a statement of the thought process by which you found your answer.

I will be checking school e-mail periodically during the summer. If I am on vacation, it may take a couple of days to get back with you, but I will.

Mr. Campbell: campbell.jb@fcboe.org

I look forward to working with you next year!

Chapter 0 Review Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–14, write an equation for the specified line.

1. through $(1, -6)$ with slope 3

2. through $(-1, 2)$ with slope $-1/2$

3. the vertical line through $(0, -3)$

4. through $(-3, 6)$ and $(1, -2)$

5. the horizontal line through $(0, 2)$

6. through $(3, 3)$ and $(-2, 5)$

7. with slope -3 and y-intercept 3

8. through $(3, 1)$ and parallel to $2x - y = -2$

9. through $(4, -12)$ and parallel to $4x + 3y = 12$

10. through $(-2, -3)$ and perpendicular to $3x - 5y = 1$

11. through $(-1, 2)$ and perpendicular to $\frac{1}{2}x + \frac{1}{3}y = 1$

12. with x-intercept 3 and y-intercept -5

13. the line $y = f(x)$, where f has the following values:

x	-2	2	4
$f(x)$	4	2	1

14. through $(4, -2)$ with x-intercept -3

In Exercises 15–18, determine whether the graph of the function is symmetric about the y-axis, the origin, or neither.

15. $y = x^{1/5}$

16. $y = x^{2/5}$

17. $y = x^2 - 2x - 1$

18. $y = e^{-x^2}$

In Exercises 19–26, determine whether the function is even, odd, or neither.

19. $y = x^2 + 1$

20. $y = x^5 - x^3 - x$

21. $y = 1 - \cos x$

22. $y = \sec x \tan x$

23. $y = \dfrac{x^4 + 1}{x^3 - 2x}$

24. $y = 1 - \sin x$

25. $y = x + \cos x$

26. $y = \sqrt{x^4 - 1}$

In Exercises 27–38, find the **(a)** domain and **(b)** range, and **(c)** graph the function.

27. $y = |x| - 2$

28. $y = -2 + \sqrt{1 - x}$

29. $y = \sqrt{16 - x^2}$

30. $y = 3^{2-x} + 1$

31. $y = 2e^{-x} - 3$

32. $y = \tan(2x - \pi)$

33. $y = 2 \sin(3x + \pi) - 1$

34. $y = x^{2/5}$

35. $y = \ln(x - 3) + 1$

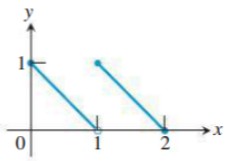
36. $y = -1 + \sqrt[3]{2 - x}$

37. $y = \begin{cases} \sqrt{-x}, & -4 \le x \le 0 \\ \sqrt{x}, & 0 < x \le 4 \end{cases}$

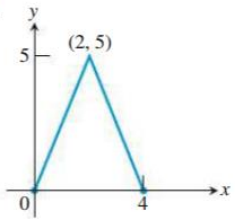
38. $y = \begin{cases} -x - 2, & -2 \le x \le -1 \\ x, & -1 < x \le 1 \\ -x + 2, & 1 < x \le 2 \end{cases}$

In Exercises 39 and 40, write a piecewise formula for the function.

39.



40.



In Exercises 41 and 42, find

(a) $(f \circ g)(-1)$ **(b)** $(g \circ f)(2)$ **(c)** $(f \circ f)(x)$ **(d)** $(g \circ g)(x)$

41. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{\sqrt{x + 2}}$

42. $f(x) = 2 - x, \quad g(x) = \sqrt[3]{x + 1}$

In Exercises 43 and 44, **(a)** write a formula for $f \circ g$ and $g \circ f$, and find the **(b)** domain and **(c)** range of each.

43. $f(x) = 2 - x^2, \quad g(x) = \sqrt{x + 2}$

44. $f(x) = \sqrt{x}, \quad g(x) = \sqrt{1 - x}$

In Exercises 45–48, a parametrization is given for a curve.

(a) Graph the curve. Identify the initial and terminal points, if any. Indicate the direction in which the curve is traced.

(b) Find a Cartesian equation for a curve that contains the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

45. $x = 5\cos t, \quad y = 2\sin t, \quad 0 \le t \le 2\pi$

46. $x = 4\cos t, \quad y = 4\sin t, \quad \pi/2 \le t < 3\pi/2$

47. $x = 2 - t, \quad y = 11 - 2t, \quad -2 \le t \le 4$

48. $x = 1 + t, \quad y = \sqrt{4 - 2t}, \quad t \le 2$

In Exercises 49–52, give a parametrization for the curve.

49. the line segment with endpoints $(-2, 5)$ and $(4, 3)$

50. the line through $(-3, -2)$ and $(4, -1)$

51. the ray with initial point $(2, 5)$ that passes through $(-1, 0)$

52. $y = x(x - 4), x \le 2$

Group Activity In Exercises 53 and 54, do the following.

(a) Find f^{-1} and show that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.

(b) Graph f and f^{-1} in the same viewing window.

53. $f(x) = 2 - 3x$

54. $f(x) = (x + 2)^2, \quad x \ge -2$

In Exercises 55 and 56, find the measure of the angle in radians and degrees.

55. $\sin^{-1}(0.6)$

56. $\tan^{-1}(-2.3)$

57. Find the six trigonometric function values of $\theta = \cos^{-1}(3/7)$. Give exact answers.

58. Solve the equation $\sin x = -0.2$ in the following intervals.

(a) $0 \le x < 2\pi$ **(b)** $-\infty < x < \infty$

59. Solve for x: $e^{-0.2x} = 4$

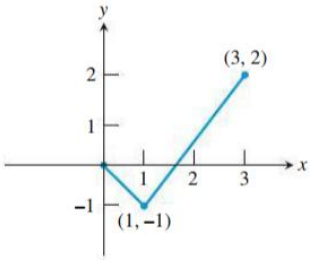
60. The graph of f is shown. Draw the graph of each function.

(a) $y = f(-x)$

(b) $y = -f(x)$

(c) $y = -2f(x + 1) + 1$

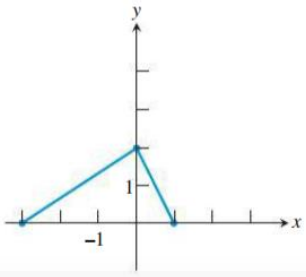
(d) $y = 3f(x - 2) - 2$



61. A portion of the graph of a function defined on $[-3, 3]$ is shown. Complete the graph assuming that the function is

(a) even.

(b) odd.



62. ***Depreciation*** Smith Hauling purchased an 18-wheel truck for $100,000. The truck depreciates at the constant rate of $10,000 per year for 10 years.

(a) Write an expression that gives the value y after x years.

(b) When is the value of the truck $55,000?

63. ***Drug Absorption*** A drug is administered intravenously for pain. The function

$$f(t) = 90 - 52\ln(1 + t), \quad 0 \le t \le 4$$

gives the number of units of the drug in the body after t hours.

(a) How many units of the drug were initially administered?

(b) How much is present after 2 hours?

(c) Draw the graph of f.

64. ***Finding Time*** If Joenita invests $1500 in a retirement account that earns 8% compounded annually, how long will it take this single payment to grow to $5000?

65. ***Guppy Population*** The number of guppies in Susan's aquarium doubles every day. There are four guppies initially.

(a) Write the number of guppies as a function of time t.

(b) How many guppies were present after 4 days? after 1 week?

(c) When will there be 2000 guppies?

(d) **Writing to Learn** Give reasons why this might not be a good model for the growth of Susan's guppy population.

66. ***The Rule of 70*** A well-known rule in the world of finance is that an annual interest rate of $R\%$ will double an investment in approximately $70/R$ years. Assume that the money is compounded continuously so that you can use the Pe^{rt} formula.

(a) Solve the equation $Pe^{rt} = 2P$ to find t as a function of r.

(b) If $r = R\%$, write t as a function of R.

(c) If the money is not compounded continuously, the doubling time will be a little longer than the answer in (b). Approximate the typical doubling time by increasing the numerator by 1. This should explain the Rule of 70!

67. **Writing to Learn** Many people refer to the Rule of 70 (see Exercise 66) as the ***Rule of 72,*** although since this is usually less accurate. Why do you suppose some people prefer it?

You may use a graphing calculator to solve the following problems.

68. Consider the point $P(-2, 1)$ and the line L: $x + y = 2$.

(a) Find the slope of L.

(b) Write an equation for the line through P and parallel to L.

(c) Write an equation for the line through P and perpendicular to L.

(d) What is the x-intercept of L?

69. Let $f(x) = 1 - \ln(x - 2)$.

(a) What is the domain of f?

(b) What is the range of f?

(c) What are the x-intercepts of the graph of f?

(d) Find f^{-1}.

(e) Confirm your answer in part (d) algebraically.

70. Let $f(x) = 1 - 3\cos(2x)$.

(a) What is the domain of f?

(b) What is the range of f?

(c) What is the period of f?

(d) Is f an even function, an odd function, or neither?

(e) Find all the zeros of f in $\pi/2 \le x \le \pi$.

Chapter 1 Review Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–14, find the limits.

1. $\lim_{x\to -2}(x^3 - 2x^2 + 1)$

2. $\lim_{x\to -2}\dfrac{x^2+1}{3x^2-2x+5}$

3. $\lim_{x\to 4}\sqrt{1-2x}$

4. $\lim_{x\to 5}\sqrt[4]{9-x^2}$

5. $\lim_{x\to 0}\dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}$

6. $\lim_{x\to \pm\infty}\dfrac{2x^2+3}{5x^2+7}$

7. $\lim_{x\to \pm\infty}\dfrac{x^4+x^3}{12x^3+128}$

8. $\lim_{x\to 0}\dfrac{\sin(2x)}{4x}$

9. $\lim_{x\to 0}\dfrac{x\csc x+1}{x\csc x}$

10. $\lim_{x\to 0} e^x \sin x$

11. $\lim_{x\to 7/2^+}\text{int}\,(2x-1)$

12. $\lim_{x\to 7/2^-}\text{int}\,(2x-1)$

13. $\lim_{x\to\infty} e^{-x}\cos x$

14. $\lim_{x\to\infty}\dfrac{x+\sin x}{x+\cos x}$

In Exercises 15–20, determine whether the limit exists on the basis of the graph of $y = f(x)$. The domain of f is the set of real numbers.

15. $\lim_{x\to d} f(x)$

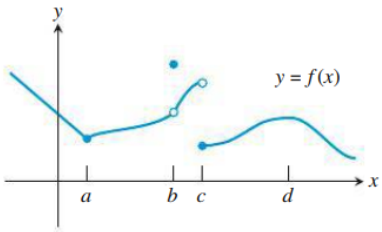
16. $\lim_{x\to c^+} f(x)$

17. $\lim_{x\to c^-} f(x)$

18. $\lim_{x\to c} f(x)$

19. $\lim_{x\to b} f(x)$

20. $\lim_{x\to a} f(x)$



In Exercises 21–24, determine whether the function f used in Exercises 15–20 is continuous at the indicated point.

21. $x = a$

22. $x = b$

23. $x = c$

24. $x = d$

In Exercises 25 and 26, use the graph of the function with domain $-1 \le x \le 3$.

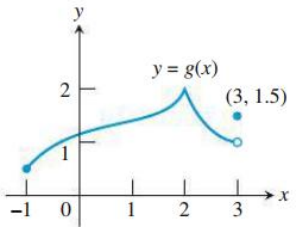
25. Determine

(a) $\lim_{x\to 3^-} g(x)$. **(b)** $g(3)$.

(c) whether $g(x)$ is continuous at $x = 3$.

(d) the points of discontinuity of $g(x)$.

(e) **Writing to Learn** Are any of these points of discontinuity removable? If so, describe the new function. If not, explain why not.



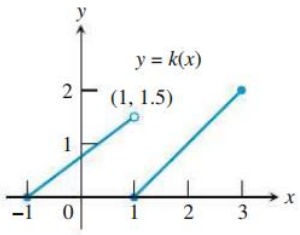
26. Determine

(a) $\lim_{x\to 1^-} k(x)$. **(b)** $\lim_{x\to 1^+} k(x)$. **(c)** $k(1)$.

(d) whether $k(x)$ is continuous at $x = 1$.

(e) the points of discontinuity of $k(x)$.

(f) **Writing to Learn** Are any of these points of discontinuity removable? If so, describe the new function. If not, explain why not.



In Exercises 27 and 28, **(a)** find the vertical asymptotes of the graph of $y = f(x)$, and **(b)** describe the behavior of $f(x)$ to the left and right of any vertical asymptote.

27. $f(x) = \dfrac{x+3}{x+2}$

28. $f(x) = \dfrac{x-1}{x^2(x+2)}$

In Exercises 29 and 30, answer the questions for the piecewise-defined function.

29. $f(x) = \begin{cases} 1, & x \le -1 \\ -x, & -1 < x < 0 \\ 1, & x = 0 \\ -x, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$

(a) Find the right-hand and left-hand limits of f at $x = -1, 0$, and 1.

(b) Does f have a limit as x approaches -1? 0? 1? If so, what is it? If not, why not?

(c) Is f continuous at $x = -1$? 0? 1? Explain.

30. $f(x) = \begin{cases} |x^3 - 4x|, & x < 1 \\ x^2 - 2x - 2, & x \ge 1 \end{cases}$

(a) Find the right-hand and left-hand limits of f at $x = 1$.

(b) Does f have a limit as $x \to 1$? If so, what is it? If not, why not?

(c) At what points is f continuous?

(d) At what points is f discontinuous?

In Exercises 31 and 32, find all points of discontinuity of the function.

31. $f(x) = \dfrac{x+1}{4-x^2}$

32. $g(x) = \sqrt[3]{3x+2}$

In Exercises 33–36, find **(a)** a power function end behavior model and **(b)** any horizontal asymptotes.

33. $f(x) = \dfrac{2x + 1}{x^2 - 2x + 1}$

34. $f(x) = \dfrac{2x^2 + 5x - 1}{x^2 + 2x}$

35. $f(x) = \dfrac{x^3 - 4x^2 + 3x + 3}{x - 3}$

36. $f(x) = \dfrac{x^4 - 3x^2 + x - 1}{x^3 - x + 1}$

In Exercises 37 and 38, find **(a)** a right end behavior model and **(b)** a left end behavior model for the function.

37. $f(x) = x + e^x$

38. $f(x) = \ln|x| + \sin x$

Group Activity In Exercises 39 and 40, what value should be assigned to k to make f a continuous function?

39. $f(x) = \begin{cases} \dfrac{x^2 + 2x - 15}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$

40. $f(x) = \begin{cases} \dfrac{\sin x}{2x}, & x \neq 0 \\ k, & x = 0 \end{cases}$

Group Activity In Exercises 41 and 42, sketch a graph of a function f that satisfies the given conditions.

41. $\lim_{x\to\infty} f(x) = 3$, $\lim_{x\to-\infty} f(x) = \infty$, $\lim_{x\to3^+} f(x) = \infty$, $\lim_{x\to3^-} f(x) = -\infty$

42. $\lim_{x\to2} f(x)$ does not exist, $\lim_{x\to2^+} f(x) = f(2) = 3$

43. *Average Rate of Change* Find the average rate of change of $f(x) = 1 + \sin x$ over the interval $[0, \pi/2]$.

44. *Rate of Change* Find the instantaneous rate of change of the volume $V = (1/3)\pi r^2 H$ of a cone with respect to the radius r at $r = a$ if the height H does not change.

45. *Rate of Change* Find the instantaneous rate of change of the surface area $S = 6x^2$ of a cube with respect to the edge length x at $x = a$.

46. *Slope of a Curve* Find the slope of the curve $y = x^2 - x - 2$ at $x = a$.

47. *Tangent and Normal* Let $f(x) = x^2 - 3x$ and $P = (1, f(1))$. Find **(a)** the slope of the curve $y = f(x)$ at P, **(b)** an equation of the tangent at P, and **(c)** an equation of the normal at P.

48. *Horizontal Tangents* At what points, if any, are the tangents to the graph of $f(x) = x^2 - 3x$ horizontal? (See Exercise 47.)

49. *Sensitivity* A ball is thrown straight up with an initial velocity of v feet per second. The maximum height is $H = v^2/64$ feet. Find and interpret the sensitivity of the height to the initial velocity when the initial velocity is 30 ft/sec.

50. *Sensitivity* An error in the measurement of the radius of a circle results in an error in the computation of its area. Find and interpret the sensitivity of the area, A, of a circle to the measurement of its radius, r, when the radius is 2 meters.

51. *Bear Population* The number of bears in a federal wildlife reserve is given by the population equation

$$p(t) = \frac{200}{1 + 7e^{-0.1t}},$$

where t is in years.

(a) Writing to Learn Find $p(0)$. Give a possible interpretation of this number.

(b) Find $\lim_{t\to\infty} p(t)$.

(c) Writing to Learn Give a possible interpretation of the result in part (b).

52. *Taxi Fares* Bluetop Cab charges \$3.20 for the first mile and \$1.35 for each additional mile or part of a mile.

(a) Write a formula that gives the charge for x miles with $0 \le x \le 20$.

(b) Graph the function in (a). At what values of x is it discontinuous?

55. Free Response Let $f(x) = \dfrac{x}{|x^2 - 9|}$.

(a) Find the domain of f.

(b) Write an equation for each vertical asymptote of the graph of f.

(c) Write an equation for each horizontal asymptote of the graph of f.

(d) Is f odd, even, or neither? Justify your answer.

(e) Find all values of x for which f is discontinuous and classify each discontinuity as removable or nonremovable.

56. Free Response Let $f(x) = \begin{cases} x^2 - a^2x & \text{if } x < 2, \\ 4 - 2x^2 & \text{if } x \ge 2. \end{cases}$

(a) Find $\lim_{x\to2^-} f(x)$.

(b) Find $\lim_{x\to2^+} f(x)$.

(c) Find all values of a that make f continuous at 2. Justify your answer.

57. Free Response Let $f(x) = \dfrac{x^3 - 2x^2 + 1}{x^2 + 3}$.

(a) Find all zeros of f.

(b) Find a right end behavior model $g(x)$ for f.

(c) Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} \dfrac{f(x)}{g(x)}$.